Simulation-Based Inference for the Synchronization of Business Cycles

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Abstract
The synchronization of business cycles has again become a topic of interest after restrictions due to the global pandemic and supply-chain issues have highlighted some risks of integration. The business cycle literature includes different methodologies to determine the degree of synchronization of business cycles across different economies. However, few business cycle synchronization hypothesis testing procedures are available. Those that are can sometimes give mixed results, require assumptions about the functional form of the relationship, and are not formally shown to be valid testing procedures, especially in finite samples. In this paper, we identify business cycles using Markov switching models as in the related literature but propose using the Monte Carlo likelihood ratio testing procedures first introduced in Rodriguez-Rondon and Dufour, 2023a and Rodriguez-Rondon and Dufour, 2022 to test the hypothesis of synchronized business cycles against different alternatives of non-synchronized business cycles. Particularly, the Local Monte Carlo (LMC) and the Maximized Monte Carlo (MMC) likelihood ratio tests, which was shown to be valid in finite samples, are proposed for this purpose. We test the hypothesis of synchronized business cycles between the Unites States and Canada, United Kingdom, or Germany using industrial production and real GDP data from 1985-Q1 to 2022-Q2. Results suggest that, when considering industrial production and real GDP data up to 2019-Q4 (pre-COVID period), the US business cycle and that of the three other economies are perfectly synchronized. However, when we include the COVID period, our results suggest that the US business cycle is no longer synchronized with that of Canada when considering industrial production data and is no longer synchronized with all three economies when considering real GDP data. These results provide some evidence suggesting that the recovery, following the pandemic, were different for these economies when compared to that of the US economy.

Key Words: Business cycles synchronization, Markov switching, Hypothesis testing, Monte Carlo, Regimes, Nonlinearity

1. Introduction

The econometrics literature contains many empirical studies which highlight the need to consider structural breaks or regime changes when working with macroeconomic and financial variables. A common example is the reduction in variance of many macroeconomic variables following the Great Moderation and the difference in mean growth rates between recessionary and expansionary regimes. Other examples include high or low volatility regimes in financial markets and passive or active monetary policy regimes. Markov switching models are often used to identify these regimes and hence the state of the economy, but a typical assumption is that only two regimes are needed to model these macroeconomic or financial variables. This is especially the case when considering multivariate (e.g., VAR(p)) models (see for example Primiceri, 2005). Part of the reason for this simplifying assumption is twofold. First, business cycles are typically characterized by a low and high period suggesting we only need two regimes by definition. However, it is not always the case that all recessionary or expansionary periods look the same or occur at the same time across the different variables, representing different economies, in the system. Second, at least until recently, a valid testing procedure to determine the appropriate number of regimes in a multivariate Markov switching model was not available.

Different methodologies have been proposed in the literature to determine whether business cycles are perfectly synchronized across different economies, whether they are independent, or to what degree these business cycles are related. In Phillips, 1991, the author considers the transmission of business cycles between two economies. To do this, he assumes that each country has two regimes, one where the economy is growing and another where the economy is in a recessionary state. To asses the transmission of business cycles between two economies, the author estimates a bivariate Markov switching model with four regimes. In one regime both economies are growing, in the second both are in recessionary states, in the third regime one economy is growing while the other is in a recessionary state, and in the fourth regime the states of each economy from the third regime are reversed. Hypothesis tests are then developed based on the resulting (4×4) transmission matrix to determine if the states of the economy are either perfectly correlated or independent from one another. When independent, the study also considers tests to determine if one country leads (lags) the other. However, in some cases, such as the one where the U.S. and U.K. economy are considered, the null hypothesis of independence and perfect correlation both cannot be rejected and as a result this testing procedure leads to mixed results. Other authors such as Guha and Banerji, 1999 and Artis, 2004 instead estimate univariate Markov switching models for each economy and compute the cross-correlations between the smoothed probabilities of being in a recession to get a measure of synchronization. However, Camacho and Perez-Quiros, 2006 use simulations to show that this method of assessing synchronization can be biased towards showing relatively low values of synchronization specifically for countries that exhibit synchronized cycles. As a result, Camacho and Perez-Quiros, 2006 and Bengoechea et al., 2006 propose estimating a bivariate Markov switching model with an additional parameter, δ_{ab}, that captures the degree of desynchronization. However, this is done by defining the probabilities of each state in the multivariate model to be a linear combination of the unsynchronized and perfectly synchronized cases, which are defined based on the probabilities from univariate models. In addition, Camacho and Perez-Quiros, 2006 also provides results from a simulation-based test procedure to asses whether business cycles are perfectly synchronized ($H_0 : \delta_{ab} = 0$) or unsynchronized ($H_0 : \delta_{ab} = 1$). Although it is not formally presented this way, this procedure is very similar to that of a Bootstrap test or the Local Monte Carlo test described in Dufour, 2006. Leiva-Leon, 2014 extends the approach of Camacho and Perez-Quiros, 2006 to state-space representation and uses Bayesian methods to estimate the models. Leiva-Leon, 2017 later extends the work of Leiva-Leon, 2014 to allow the degree of synchronization to be time-varying, while also using Bayesian methods, allowing them to investigate the interdependency of business cycles of US states. One of the shortcomings of these recent methods however, is that they all only allow for the Markov processes governing each variable to have two regimes, whereas it may be possible that some variables, although still subject to expansionary and recessionary regimes, have a Markov process that has more than two regimes. For example, there can be relatively small and large recession and expansionary regimes which would result in up to four regimes. In the case of macroeconomic variables, we may find that there are indeed only recessionary and expansionary regimes but that the variance of the expansionary regime was higher before the mid 1980s compared to after (i.e. Great Moderation), leading to three regimes. Further, with regards to the testing procedure discussed in Camacho and Perez-Quiros, 2006, there is no formal results showing power and size of this test for different values of δ_{ab} and it is not clear how this test performs in finite samples, which is often the case when considering macroeconomic variables.

In this paper, we consider using the Monte Carlo Likelihood Ratio testing procedures for Markov switching models first introduced in Rodriguez-Rondon and Dufour, 2022 and Rodriguez-Rondon and Dufour, 2023a. These testing procedures can be used to determine the appropriate number of regimes in a system. To test for perfectly correlated business cycles against different alternatives of independent business cycles when considering two economies, we test for the number of regimes of the Markov process governing a bi-variate model. By using this methodology, we are able to let the data speak and can consider cases where each variable is subject to more than one type of recessionary or expansionary regime if necessary. We can also consider different types of independence not considered previously in the literature. For example, in the case where each variable has only two regimes and these variables enter (exit) a state at the same time but exit (enter) that state at different time, the appropriate alternative is a model with three regimes instead of four. The methodology proposed in Rodriguez-Rondon and Dufour, 2022 and Rodriguez-Rondon and Dufour, 2023a is general enough to consider both these different scenarios and the Maximized Monte Carlo version was shown to be valid even in finite samples, which is especially important when considering more complicated models with many regimes as shown in Rodriguez-Rondon and Dufour, 2023a.

As in Phillips, 1991 we consider industrial production data for the United States, Canada, United Kingdom, and Germany at the quarterly frequency. Since industrial production only covers the supply side of the economy, we also consider real GDP data as in Camacho and Perez-Quiros, 2006. Throughout our analysis we consider two samples. The first ends in 2019Q4 before the global pandemic. The second sample ends in 2022 Q2 and hence includes the COVID-19 period. Both samples begin in Q1 of 1985, when the low volatility period following the Great Moderation begins. When using the first sample we find that, for industrial production, there is strong evidence that the business cycle in Canada, UK, and Germany is perfectly correlated with that of the US, suggesting there is a single global business cycle. When using GDP data, we find similar results with the exception that the US and UK do not have perfectly correlated business cycles. When we extend the sample to include the COVID-19 period, we find that the US business cycle is no longer synchronized with the Canadian business cycle, when considering industrial production data and no longer synchronized with any economy when considering real GDP data.

As a result, this study presents a simulation-based hypothesis testing procedure for testing the hypothesis of perfectly correlated business cycles that is valid both asymptotically and in finite samples. We also document evidence that the including the COVID-19 period has consequences for the synchronization of the business cycles considered here. Still, there is more work that needs to be done in terms of correcting the model to better deal with the COVID-19 period, which many agree can be understood as an outlier problem. For this reason, this is the subject of ongoing work.

The next sections are structured as follows. Section 2 reviews the Markov switching autoregressive and vector autoregressive models that we use for our analysis. In section 3, we discuss the MMC-LRT and LMC-LRT for MS-VAR models proposed Rodriguez-Rondon and Dufour, 2023a. Section 4 describes the data we use for our analysis and section 5 discusses the results of our testing procedure and how it ties to our finding of only one international business cycle. Finally, section 5 provides concluding remarks and directions for future research.

2. Markov switching

In general, a Markov switching model can be expressed as

$$y_t = x_t\beta + z_t\delta_{s_t} + \sigma_{s_t}\epsilon_t\,. \tag{1}$$

In a univariate setting, y_t is a scalar, x_t is a $1 \times n$ vector of variables whose coefficients do not depend on the latent Markov process S_t, z_t is an $1 \times \nu$ vector of variables whose coefficient depend on the Markov process S_t, and ϵ_t is an error process, which for example may follow a $\mathcal{N}(0,1)$ distribution, and σ_{s_t} a standard deviation which may also depend on the Markov process S_t or remain constant throughout (*i.e.*, σ). Here, we will assume that explanatory variables are fixed (or predetermined). For our testing procedures, other distributions on the error processes may be considered, but for simplicity we assume a normal distribution. In order to have an autoregressive MSM, lags of y_t are included in either x_t or z_t depending on whether we want to allow the autoregressive coefficients to depend on the regimes. A Hidden Markov model can also be recovered by considering only a constant term in z_t and excluding x_t. In a multivariate setting, we allow y_t to be a $1 \times q$ vector (*i.e.*, $y_t = (y_{1,t}, \ \ldots \ , y_{q,t})$) and ϵ_t also a $1 \times q$ vector, which may be distributed as $\mathcal{N}(\mathbf{0}, \Sigma_{s_t})$ or $\mathcal{N}(\mathbf{0}, \Sigma)$ (if the variance-covariance matrix does not depend on the latent Markov process S_t). For example, a bivariate Markov-switching VAR model is given by

$$\mathbf{y}_t = \boldsymbol{\mu}_{S_t} + \Phi_1(\mathbf{y}_t - \boldsymbol{\mu}_{S_{t-1}}) + \cdots + \Phi_p(\mathbf{y}_{t-p} - \boldsymbol{\mu}_{S_{t-p}}) + \boldsymbol{\epsilon}_t \tag{2}$$

where $\boldsymbol{y_t} = (y_{1,t}, y_{2,t})'$, $\boldsymbol{\mu}_t = (\mu_{1,S_t}, \mu_{2,S_t})'$, $\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \epsilon_{2,t})'$, and Φ_k is a (2×2) matrix containing the autoregressive parameters and lag k.

As described in Hamilton (1994), for a model with M regimes, the one-step transition probabilities can be gathered into a transition matrix such as

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{M1} \\ \vdots & \ddots & \vdots \\ p_{1M} & \cdots & p_{MM} \end{bmatrix} \tag{3}$$

where for example $p_{ij} = P(S_t = j \,|\, S_{t-1} = i)$ is the probability that state i switches to state j. For example, if we consider a model with only two regimes, we only need a 2×2 transition matrix to summarize the transition probabilities $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} \tag{4}$$

In either case, the columns of the transition matrix must sum to one in order to have a well defined transition matrix (*i.e.*, $\sum_{j=1}^{M} = p_{ij} = 1$). We can also obtain the ergodic probabilities, $\pi = (\pi_1, \pi_2)'$, which are given by

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}\,, \quad \pi_2 = 1 - \pi_1\,, \tag{5}$$

in a setting with two regimes or, more generally, for any number of M regimes we could use

$$\boldsymbol{\pi} = (\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{e}_{N+1}\,, \quad \mathbf{A} = \begin{bmatrix} \mathbf{I}_M - \mathbf{P} \\ \mathbf{1}' \end{bmatrix}\,, \tag{6}$$

where $\mathbf{e}_{M+1}$ is the $(M+1)$-th column of $\mathbf{I}_{M+1}$.

Continuing with the example of a MSM such as the one given by (2) with $S_t = \{1, 2\}$ (*i.e.*, $M = 2$ regimes), the log-likelihood conditional on the first p observations of $\boldsymbol{y}_t$ is given by

$$L_T(\theta) = \log f(\boldsymbol{y}_1^T \,|\, \boldsymbol{y}_{-p+1}^0;\, \theta) = \sum_{t=1}^{T} \log f(\boldsymbol{y}_t \,|\, \boldsymbol{y}_{-p+1}^{t-1};\, \theta) \tag{7}$$

where $\theta = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \mathrm{vec}(\boldsymbol{\Sigma}_1), \mathrm{vec}(\boldsymbol{\Sigma}_2), \mathrm{vec}(\boldsymbol{\Phi}_1), \ldots, \mathrm{vec}(\boldsymbol{\Phi}_p), p_{11}, p_{22})'$ and

$$f(\boldsymbol{y}_t \,|\, \boldsymbol{y}_{-p+1}^{t-1};\, \theta) = \sum_{s_t=1}^{2} \sum_{s_{t-1}=1}^{2} \cdots \sum_{s_{t-p}=1}^{2} f(\boldsymbol{y}_t,\, S_t = s_t,\, S_{t-1} = s_{t-1},\, \ldots,\, S_{t-p} = s_{t-p} \,|\, \boldsymbol{y}_{-p+1}^{t-1};\, \theta)\,. \tag{8}$$

Under Gaussianity, we have:

$$f(\boldsymbol{y}_t,\, S_t = s_t,\, \ldots,\, S_{t-p} = s_{t-p} \,|\, \boldsymbol{y}_{-p+1}^{t-1};\, \theta) = \frac{\mathbb{P}(S_t^* = s_t^* \,|\, \boldsymbol{y}_{-p+1}^{t-1};\, \theta)}{2\pi|\Sigma_{s_t}|^{1/2}} \times \exp\left\{-\frac{1}{2}[\boldsymbol{y}_t - \boldsymbol{\mu}_{s_t} - \sum_{k=1}^{p} \boldsymbol{\Phi}_k(\boldsymbol{y}_{t-k} - \boldsymbol{\mu}_{s_{t-k}})]'\boldsymbol{\Sigma}_{s_t}^{-1}[\boldsymbol{y}_t - \boldsymbol{\mu}_{s_t} - \sum_{k=1}^{p} \boldsymbol{\Phi}_k(\boldsymbol{y}_{t-k} - \boldsymbol{\mu}_{s_{t-k}})]\right\} \tag{9}$$

where

$$S_t^* = s_t^* \;\text{ if }\; S_t = s_t,\, S_{t-1} = s_{t-1},\, \ldots,\, S_{t-p} = s_{t-p} \tag{10}$$

and $\mathbb{P}(S_t^* = s_t^* \,|\, \boldsymbol{y}_{-p+1}^{t-1}; \theta)$ is the probability that this occurs.

Typically, MSMs are estimated using the Expectation Maximization (EM) algorithm [see Dempster et al., 1977], Bayesian methods or through the use of the Kalman filter [using the state-space representation of the model)]. In very simple cases, MSMs can be estimated using Maximum Likelihood Estimation (MLE). However, since the Markov process S_t is latent and more importantly the likelihood function can have several modes of equal height in addition to other unusual features that can complicate estimation by MLE this is not often used. In this study, we use the EM algorithm when estimating MSMs. It is worth noting that, in practice, empirical estimates can sometimes be improved by using the results of the EM algorithm as initial values in a Newton-type of optimization algorithm. This two-step estimation procedure is used to obtain results presented in the empirical section of this paper. We omit a detailed explanation of the EM algorithm as our focus is on the hypothesis testing procedures proposed here. For the interested reader, the estimation of a Markov switching model via the EM algorithm is describe in detail in Hamilton (1990) and Krolzig (1997).

3. Monte Carlo likelihood ratio tests for synchronization of business cycles

In this paper, we propose testing for the synchronization of two business cycles by identifying the appropriate number of regimes of a bivariate Markov switching VAR model that includes both economies being considered. Business cycles are define as intervals of recession and expansions and so we should consider a Markov process with at least two regimes. If the business cycles of both economies considered in the system are synchronized, we should expect that one Markov process with two states is sufficient to explain our data. However, if the business cycles of these economies are not synchronized then each requires their own independent Markov process to model the states of each economy. In the later case, we can still use a single Markov process, but with more states to summarize the state of each underlying Markov process. For example, consider the following bi-variate model with economies a and b

$$y_{a,t} = \mu_{a,s_{a,t}} + \sum_{k=1}^{p} \phi_{aa,k} \left(y_{a,t-k} - \mu_{a,s_{a,t-k}} \right) + \sum_{k=1}^{p} \phi_{ab,k} \left(y_{b,t-k} - \mu_{b,s_{b,t-k}} \right) + \sigma_{a,s_{a,t}} \epsilon_{a,t}$$
$$y_{b,t} = \mu_{b,s_{b,t}} + \sum_{k=1}^{p} \phi_{ba,k} \left(y_{a,t-k} - \mu_{a,s_{a,t-k}} \right) + \sum_{k=1}^{p} \phi_{bb,k} \left(y_{b,t-k} - \mu_{b,s_{b,t-k}} \right) + \sigma_{b,s_{b,t}} \epsilon_{b,t}$$

Here, we are interested in knowing if the Markov processes $S_{a,t}$ and $S_{b,t}$ are perfectly dependent such that $S_{a,t} = S_{b,t} = S_t$ or if they are independent such that $S_{a,t} \neq S_{b,t}$. If we suppose that $S_{a,t} = \{1, 2\}$ and $S_{b,t} = \{1, 2\}$ then we should consider up to four cases:

$$\begin{aligned} S_t^* &= 1 \;\text{ if }\; S_{a,t} = 1 \;\;\&\;\; S_{b,t} = 1 \\ S_t^* &= 2 \;\text{ if }\; S_{a,t} = 1 \;\;\&\;\; S_{b,t} = 2 \\ S_t^* &= 3 \;\text{ if }\; S_{a,t} = 2 \;\;\&\;\; S_{b,t} = 1 \\ S_t^* &= 4 \;\text{ if }\; S_{a,t} = 2 \;\;\&\;\; S_{b,t} = 2 \end{aligned}$$

and if the Markov processes are perfectly dependent, then we have the following two cases

$$\begin{aligned} S_t^* &= 1 \;\text{ if }\; S_{a,t} = 1 \;\;\&\;\; S_{b,t} = 1 \\ S_t^* &= 2 \;\text{ if }\; S_{a,t} = 2 \;\;\&\;\; S_{b,t} = 2 \end{aligned}$$

We can also consider a specific type of dependence where one of the Markov processes, say $S_{a,t}$ ($S_{b,t}$) is leading (lagging) the other. Here we have the following three cases

$$\begin{aligned} S_t^* &= 1 \;\text{ if }\; S_{a,t} = 1 \;\;\&\;\; S_{b,t} = 1 \\ S_t^* &= 2 \;\text{ if }\; S_{a,t} = 2 \;\;\&\;\; S_{b,t} = 1 \\ S_t^* &= 3 \;\text{ if }\; S_{a,t} = 2 \;\;\&\;\; S_{b,t} = 2 \end{aligned}$$

As a result, testing for the synchronization of business cycles boils down to testing the null hypothesis of a Markov switching model with two regimes (i.e., business cycles are synchronized) against the alternative hypothesis of three or four regimes (i.e., not synchronized). That is, we are interested in testing

$$H_0 : M_0 = 2 \quad \text{vs.}$$

$$H_{1a} : M_0 + m = 3 \quad \text{or} \quad H_{1b} : M_0 + m = 4$$

where M_0 is the number of regimes for a bivariate Markov switching model under the null hypothesis and $M_0 + m$ is the number of regimes under the alternative.

Due to violations of regularity conditions and unidentified nuisance parameters conventional hypothesis testing procedures are not valid when comparing Markov switching models with different numbers of regimes. The econometrics literature includes some alternative hypothesis testing procedures for determining the number of regimes in a Markov switching model but most are limited to comparing only one regime under the null hypothesis against an alternative of two regimes. As a result, for our purposes, we use the Monte Carlo Likelihood Ratio test procedures proposed by Rodriguez-Rondon and Dufour, 2023a and Rodriguez-Rondon and Dufour, 2022. Another possible choice from the literature includes the parametric bootstrap procedure discussed in Kasahara and Shimotsu, 2018 as it is the only other test available in the literature that also allows for a null hypothesis of $M_0 = 2$ or more regimes. However, Kasahara and Shimotsu, 2018 only consider an alternative of $M_0 + 1$ regimes and hence cannot be used to consider the alternative of independent business cycles. Further, the Local Monte Carlo version of the procedure proposed by Rodriguez-Rondon and Dufour, 2023a is essentially like a parametric bootstrap, but the authors show that this test can be used under less restrictive assumptions and can be used to consider alternative hypotheses of $M_0 + m$ regimes where $m \geq 1$, which as discussed is of interest in our setting. Finally, the Monte Carlo Likelihood Ratio test procedure proposed by Rodriguez-Rondon and Dufour, 2023a includes the Maximized Monte Carlo version which is valid even in finite samples, which is important given that we typical only have variables measured at a quarterly frequency when analyzing business cycles. For a more detailed description of the Monte Carlo likelihood ratio testing procedures, their assumptions, implementation, and simulation results we refer the interested reader to the above mentioned studies, which discuss this testing procedure in more detail.

4. Data

In this section we describe the data used to illustrate the use of the Monte Carlo likelihood ratio test for testing the synchronization of two business cycles. We consider the same economies as in Phillips, 1991, namely, the U.S., Canada, Germany, and the U.K. As in Phillips, 1991, we also consider seasonally adjusted industrial production data at the quarterly frequency. Figure 1 and 2 show the data for each country for each sample considered here. We separate these two figures as the large variation observed during the COVID-19 period makes it difficult to see the variation for the pre-COVID-19 period.

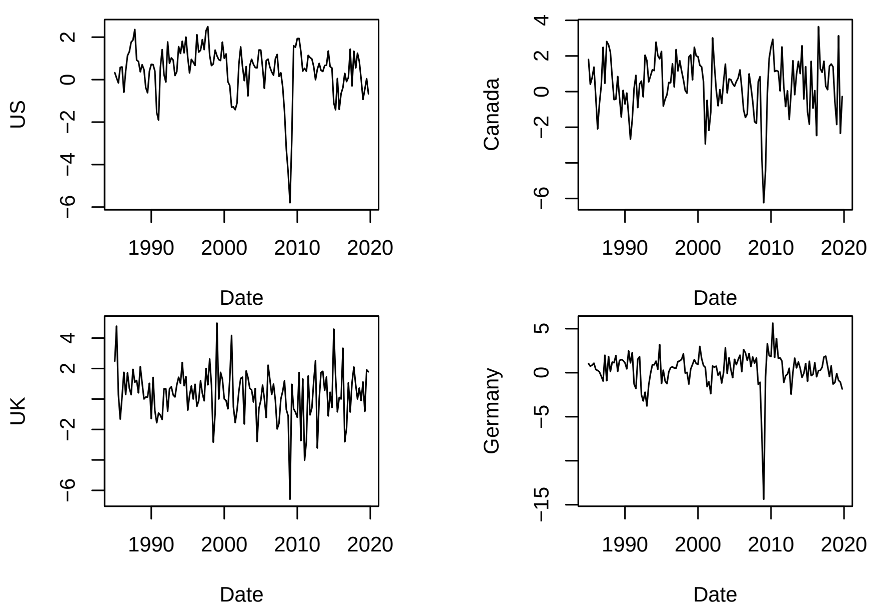

Figure 1: Industrial production 1985Q1 - 2019Q4

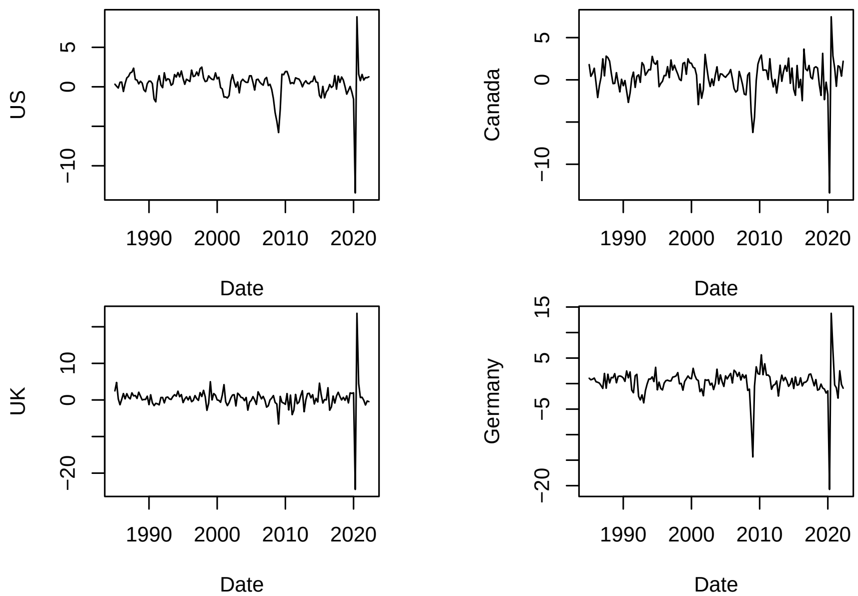

Figure 2: Industrial production 1985Q1 - 2022Q2

Since industrial production only covers the supply side of the economy, we also consider seasonally adjusted real GDP data at the quarterly frequency as in Camacho and Perez-Quiros, 2006. The two samples considered in this study for each country are shown in figure 3 and 4

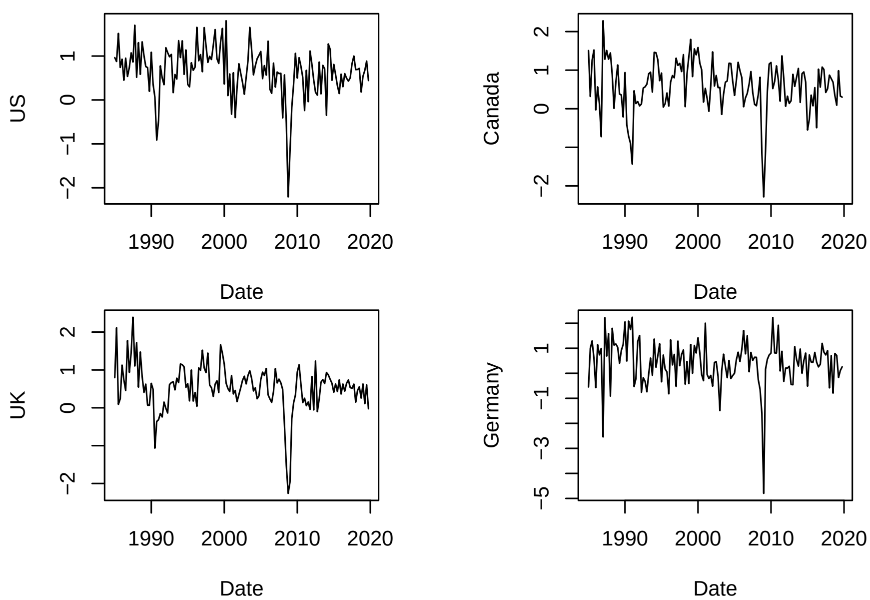

Figure 3: real GDP 1985Q1 - 2019Q4

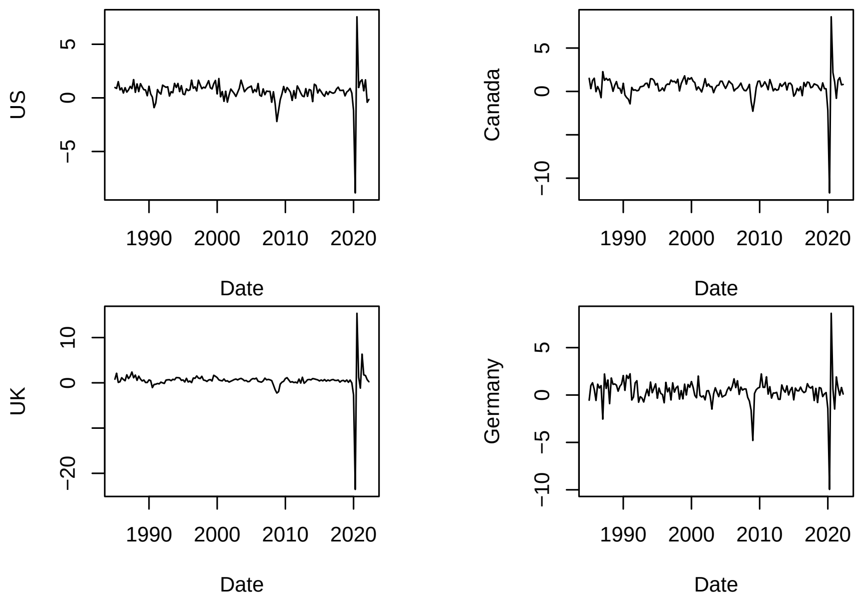

Figure 4: real GDP 1985Q1 - 2022Q2

As can be seen from all figures that include the full sample, the COVID-19 period introduces very large variations that are short-lasting. The econometrics literature includes studies discussing different ways to deal with this issue as results may be driven by what many are now considering to be an outlier-type of event. For now, we do not deal with this period in any special way and perform our analysis using the data as is. It is important to note however, that our results are subject to not treating the data *a priori* in any way. An analysis of the robustness of our results to different ways of dealing with this period is of interest and will be included in future work.

5. Empirical Results

Here we specifically show results when using the Local Monte Carlo likelihood ratio test (LMC-LRT) proposed in Rodriguez-Rondon and Dufour, 2023a on bi-variate models with the following combination of economies: (1) US-Canada, (2) US-UK, and (3) US-Germany. We will consider the null hypothesis of $H_0 : M_0 = 1$ against the alternative hypothesis of $H_a : M_0 + m = 2$ to show that there are at least two regimes that must be considered and hence that the assumption that these economies experience business cycles is valid. Next, to test the hypothesis of perfectly synchronized business cycles, we will test the null hypothesis of $H_0 : M_0 = 2$ against the alternative hypothesis of $H_1 : M_0 + m = 3$ and $H_1 : M_0 + m = 4$. The Maximized Monte Carlo likelihood ratio test (MMC-LRT) results, other simulation results, as well as other robustness checks can be found in Rodriguez-Rondon and Dufour, 2023c which is a more complete version of this work. It is also worth noting that all results were computed using the **MSTest** R-package described in Rodriguez-Rondon and Dufour, 2023b.

5.1 Multivariate model test results

The LMC-LRT results in the first panel of table 1, which considers quarterly seasonally adjusted industrial production data and does not include the COVID-19 period, suggest that the Canadian, German, and UK business cycles are perfectly synchronized with that of the U.S. since we find that each bi-variate system has only two regimes.

Table 1: Industrial production: Monte Carlo Tests - Bivariate VAR(1)

Monte Carlo P-Values of LMC-LRT			
Country	$H_0 : M = 1$ vs. $H_1 : M = 2$	$H_0 : M = 2$ vs. $H_1 : M = 3$	$H_0 : M = 2$ vs. $H_1 : M = 4$
1985-Q1 to 2019-Q4 ($T = 140$)			
US-Canada	0.01	0.19	0.23
US-UK	0.01	0.18	0.21
US-Germany	0.01	0.58	0.76
1985-Q1 to 2022-Q2 ($T = 150$)			
US-Canada	0.01	0.05	0.03
US-UK	0.01	0.18	0.12
US-Germany	0.01	0.19	0.14

In the second panel when the COVID-19 period is included however, there is some evidence that the US and Canadian business cycles are no longer perfectly synchronized as we reject the null hypothesis of two regimes.

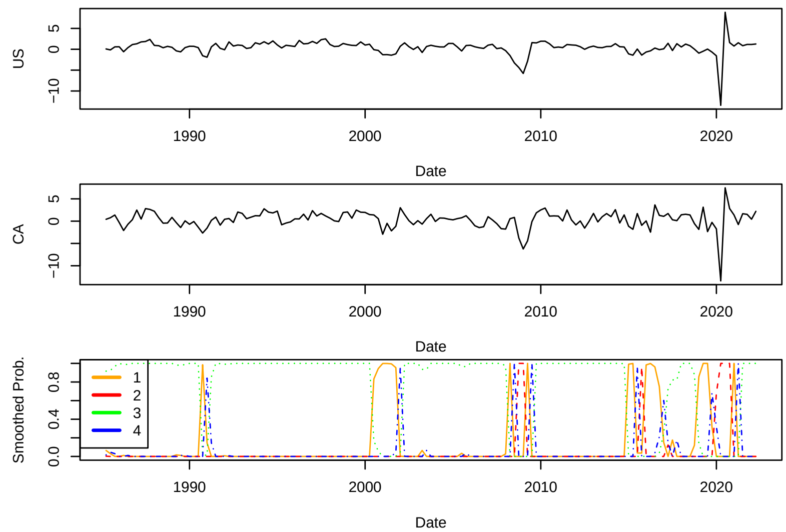

Figure 5: U.S. & Canada Industrial production & Smoothed Probabilities

These results combined with the smoothed probabilities shown in the third panel of figure 5 suggest that the recovery following the COVID-19 pandemic of these two economies were at the very least statistically different given that we now have a regime (in blue) where the U.S. industrial production has recovered but the Canadian industrial production is lagging and has not yet recovered.

Table 2: Real GDP: Monte Carlo Tests - Bivariate VAR(1)

Monte Carlo P-Values of LMC-LRT			
Country	$H_0 : M = 1$ vs. $H_1 : M = 2$	$H_0 : M = 2$ vs. $H_1 : M = 3$	$H_0 : M = 2$ vs. $H_1 : M = 4$
1985-Q1 to 2019-Q4 ($T = 140$)			
US-CA	0.02	0.20	0.17
US-UK	0.01	0.01	0.01
US-GR	0.03	0.27	0.11
1985-Q1 to 2022-Q2 ($T = 150$)			
US-CA	0.01	0.08	0.03
US-UK	0.01	0.13	0.01
US-GR	0.01	0.21	0.04

Next, we use the LMC-LRT procedure on real GDP data. These results are shown in table 2 above. Here, we find that the U.S. and UK economies are not perfectly synchronized in either sample whereas the U.S. economy appears to be perfectly synchronized with that of Canada and Germany in the first sample when we do not include the COVID-19 period. On the other hand, when the sample is extended there is no evidence that the economies are perfectly synchronized anymore.

6. Conclusion

This paper shows how the Monte Carlo likelihood ratio tests described in Rodriguez-Rondon and Dufour, 2023a and Rodriguez-Rondon and Dufour, 2022 can be used as a valid testing procedure for testing the synchronization of business cycles. Specifically, these Monte Carlo testing procedures are shown to be valid asymptotically and in finite samples, which is relevant to many macroeconomic applications including the one considered here. In order to illustrate the use of these testing procedures in the context of testing for the synchronization of business cycles, we consider three bi-variate systems, namely for US-Canada, US-Germany, and US-UK while using quarterly seasonally adjusted industrial production and real GDP data. When considering industrial production and real GDP data up to 2019-Q4 (i.e., excluding COVID-19 period), the US business cycle and that of the three other economies are perfectly synchronized. However, when we include the COVID period, our results suggest that the US business cycle is no longer synchronized with that of Canada when considering industrial production data and is no longer synchronized with all three economies when considering real GDP data. These results provide some evidence suggesting that the recovery, following the pandemic, were different for these economies when compared to that of the US economy. Interesting directions for future research include performing robustness checks when considering different methods to deal with the COVID period as suggested in recent literature. Further, it may be of interest to formalize testing procedure in Camacho and Perez-Quiros, 2006 for the degree of synchronization and propose a MMC version. This may be useful after the procedure proposed here is used to test for synchronization against no synchronization. Such topics are the subject of ongoing work.

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