Underlying Core Inflation with Multiple Regimes

Gabriel Rodriguez-Rondon

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Objective

Propose a method to improve indicators of core inflation built using factor models by considering multiple inflation regimes (e.g., high vs. low).

- Some central banks use the common factor among many disaggregate prices indices to build a measure of core inflation
- Current and recent levels of inflation in many countries, including Canada, are markedly different from previous period
- Improve signal of underlying inflation & reduce revisions

Core Inflation Indicators

Inflation-targeting central banks use various types of indicators to asses inflationary pressure that are robust to:

1. high frequency volatility from transitory shocks
2. sector-specific changes (focusing on overall inflation)

and are hence better short-term guides for monetary policy.

For example, these include:

- CPI less most volatile items [1]
- weighted CPIs [1]
- median CPI [1]
- common factor [1,2]

*Note: Superscripts indicate features that do not affect those indicators.

Example: Bank of Canada core CPI measures

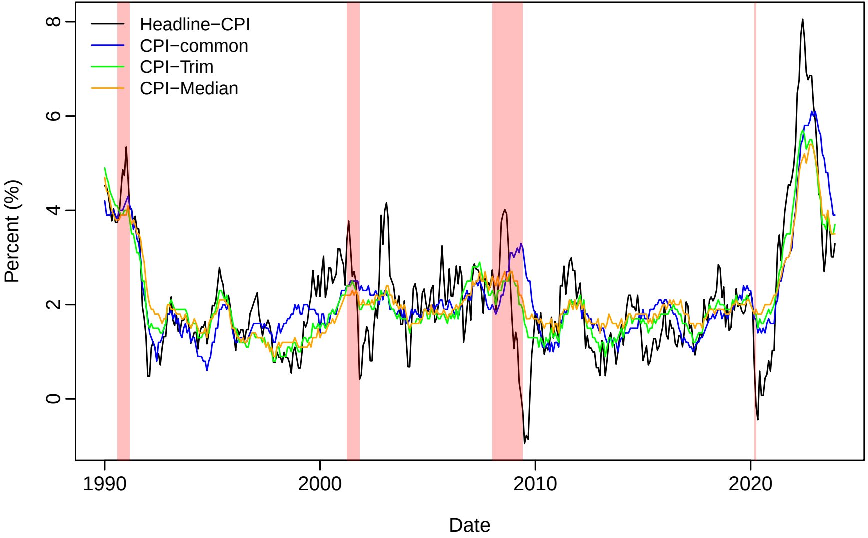

Figure 1: BoC core inflation measures from Jan-1990 to Dec-2023 (Monthly)

Core Inflation Indicators - Factor Models

Factor models for core inflation:

- Bank of Canada computes CPI-Common (see Khan et al. (2013); Khan et al. (2015))
- U.S. Fed computes Underlying Inflation Gauge (UIG) (see Amstad and Potter (2009); Amstad (2017)) and Multivariate Common Inflation Trend (based on Stock and Watson (2016))
- Others include UK (see Kapetanios (2004)), Euro area (see Cristadoro et al. (2005)), New Zealand (see Giannone and Matheson (2007); Kirker (2010)), and Turkey (see Tekatlı (2010))

Also used for:

- Now-casting Macroeconomic variables in real-time (see Banbura et al. (2013))

Issues with Factor Models for Core Inflation

Issues with these indicators include:

- subject to revisions every time new data becomes available
  - Historically, revisions have not been very large but recently this is no longer true
- must choose number of factors (typically assumed to be one for inflation; alternatively, can use Bai and Ng (2002) in some cases)

BoC CPI-Common: Worst Revision in Sample

The April vs. December 2022 CPI-Common was subject to a revision of about 2.47%

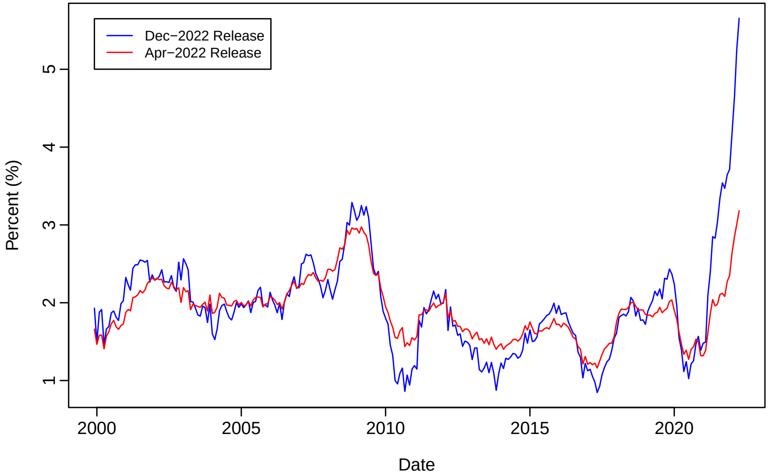

Figure 2: Worst BoC CPI-Common Revision in sample Jan-1990 to Dec-2022

BoC CPI-Common

$$\pi_t = \alpha + \beta \hat{F}_t + \epsilon_t \tag{1}$$

Recently, Sullivan (2022) showed that these larger revisions are due to three main sources:

1. revisions to the mean of inflation (α)
2. revisions to the common factor ($\hat{F}$)
3. revisions to the sensitivity of CPI to the common factor (β)

with 2 and 3 being the largest contributors

As a result, the BoC is reassessing the use of CPI-Common (see Macklem (2022)).

Contributions

This paper proposes estimating underlying core inflation while considering multiple regimes.

- this indicator
  - has the desirable features of a core inflation indicator
  - ⋆ is robust to abrupt changes
  - ⋆ provides a better signal of underlying inflationary pressure (fewer/smaller revisions)
  - ⋆ Markov Switching: useful in real-time/as short-term guide for monetary policy
- identify dates when changes in regime occur in the common factor of underlying core inflation indicators
- contribute to ongoing discussion regarding measuring underlying inflation during different inflation regimes

Core inflation with multiple regimes

$$\pi_t = \alpha + \beta_j \tilde{F}_t + \epsilon_t \tag{2}$$

where $\tilde{F}_t$ is a $r_j \times 1$ estimated from

$$X_t = \lambda_j F_t + e_t, \text{ if } z_t = j, \text{ for } t = 1, \dots, T \tag{3}$$

where π_t is a measure of headline inflation, $X_t = (x_{1t}, \dots, x_{Nt})'$, $\lambda_j = (\lambda_{j1}, \dots, \lambda_{jN})'$, and $e_t = (e_{1t}, \dots, e_{Nt})'$. Predicted values $\hat{\pi}_t$ are the resulting underlying core inflation indicator with multiple regimes.

model is robust to changes in (2) common factor and (3) the sensitivity to the common factor (β), which are shown to contribute to large revision (Sullivan (2022)). For (1) changes in mean, can consider α_j by imposing same change dates as in common factor(s), which can be tested using conventional testing procedures.

Structural Change vs. Markov Switching for z_t

Structural Change

- detect multiple break dates (can have many types of regimes)
- fewer/no assumption about process governing regime changes
- well documented hypothesis testing procedures
- Estimation: LS or QML (see Baltagi et al. (2021) and Duan et al. (2022))
- Can fully eliminate revisions for past regimes
- off-line method

Markov switching

- Markov process governs regime changes
- flexibility (many regimes) comes at higher computational cost
- hypothesis testing procedures are current research problems
- can have earlier detection of regime change (useful for real-time purposes)
- Estimation: EM Algorithm (see Urga and Wang (2023) and Barigozzi and Massacci (2022))

Core inflation indicator with Markov switching

$$X_t = \lambda_j F_t + e_t, \text{ if } z_t = j, \text{ for } t = 1, \dots, T \tag{4}$$

where $z_t = \{1, \dots, M\}$ is a latent Markov process, M is the number of regimes, and the one-step transition probabilities are summarized in the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{M1} \\ \vdots & \ddots & \vdots \\ p_{1M} & \cdots & p_{MM} \end{bmatrix}$$

where $p_{ij} = P(z_t = j | z_{t-1} = i)$ is the probability of state i being followed by state j. We can also obtain the ergodic probabilities, $\phi = (\phi_1, \dots, \phi_M)'$.

Estimated using EM Algorithm described in Urga and Wang (2023). Hypothesis testing for number of regimes is subject of ongoing research (see Rodriguez-Rondon and Dufour (2024)).

Data - Canada

- Same as CPI-common: 55 components of the CPI (see Appendix of Statistics Canada (2020) for full list)
- Monthly data from January 1990 to December 2023
- Series are adjusted to remove the effect of changes in indirect taxes and are expressed in year-over-year percentage changes
- Series are never revised

Markov switching

Markov switching in factor models:

- Estimate model with
  - $\hat{\pi}_t^{M1}$: benchmark - no Markov switching
  - $\hat{\pi}_t^{M2}$: Core inflation with $M = 2$ regimes
  - $\hat{\pi}_t^{M3}$: Core inflation with $M = 3$ regimes

Comparison:

1. Real-time performance
   - visual
   - real-time vs. full-information (see Khan et al. (2024))
2. Forecasting headline inflation

Estimation Results

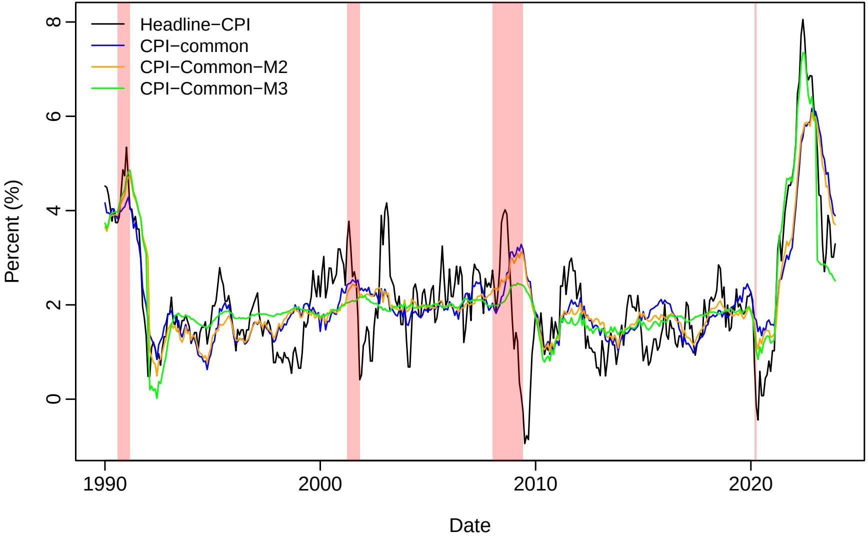

Figure 3: Canada underlying core inflation with Markov switching Jan-1990 to Dec-2023

Real-time Results

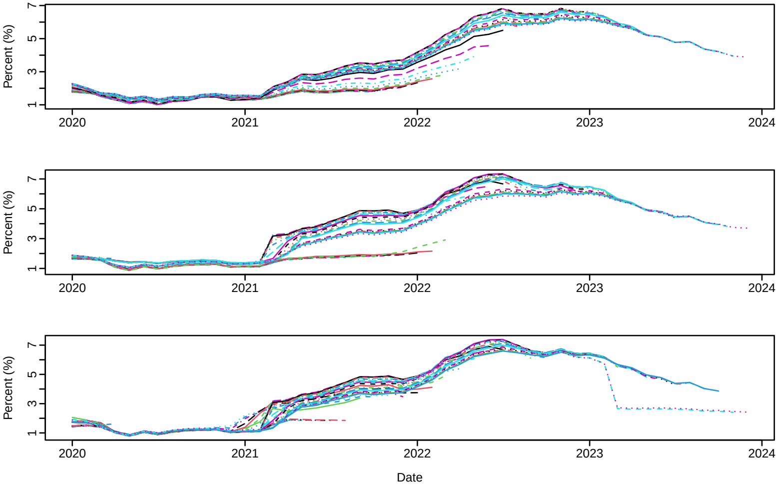

Figure 4: Canada Revisions CPI-Common Jan-2020 to Dec-2023

Real-time Results

Define

$$\hat{\pi}_t^{m,f} = E[\pi_t^m | \mathcal{I}_{T_s}] \quad (5)$$

$$\hat{\pi}_t^{m,r} = E[\pi_t^m | \mathcal{I}_t] \quad (6)$$

where $m = \{M1, M2, M3\}$, the superscript f denotes full information estimates, and superscript r denotes real-time estimates.

Table 1: Real time vs. Full Info. for each model

	Pre-Covid		**Rising Inflation**		Post-COVID		**Full-Sample**	
	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
$\hat{\pi}_t^{M1}$	**0.011**	**0.085**	1.427	0.888	**0.505**	**0.486**	**0.148**	0.273
$\hat{\pi}_t^{M2}$	0.092	0.185	2.249	1.051	0.589	0.500	0.172	**0.252**
$\hat{\pi}_t^{M3}$	0.195	0.306	**0.756**	**0.624**	1.213	0.797	0.274	0.320

Notes: MSE is Mean Squared Error, while MAE is Mean Absolute Error. In this table, we compare real-time estimates against the full information estimates for each model. That is, we use the difference $\hat{\pi}_t^{m,r} - \hat{\pi}_t^{m,f}$ for each model m and for each sample. Lowest values are highlighted.

Forecasting Results

Estimate $\hat{\pi}_t^m$ using expanding window and forecast Post-COVID period (Jan-2020 to Dec-2023).

$$\text{MSFE}_h^m = \frac{\sum_i^N (\hat{\pi}_t^m - \pi_{t+h}^{\text{HCPI}})^2}{N} \quad (7)$$

Table 2: Forecasting headline inflation π_{t+h}^{HCPI}

Models	$h = 1$	$h = 6$	$h = 12$	$h = 18$
$\hat{\pi}_t^{M1}$	3.263	6.210	**8.567**	**9.401**
$\hat{\pi}_t^{M2}$	**2.668**	**6.024**	9.344	10.390
$\hat{\pi}_t^{M3}$	**1.365**	**3.782**	**7.697**	**10.161**

Notes: Reported values are the MSFE_h^m. Estimation is performed using an expanding window. The first window ends on Dec-2019 while the last window ends on Jun-2022 and hence $N = 30$. Using the $Tmax$ test of Hansen et al. (2011) with $\alpha = 0.25$. Values in bold highlight models that belong to $\hat{\mathcal{M}}_{75\%}$ and values in blue highlight those that belong to the MCS and have lowest MSFE.

Forecasting Results

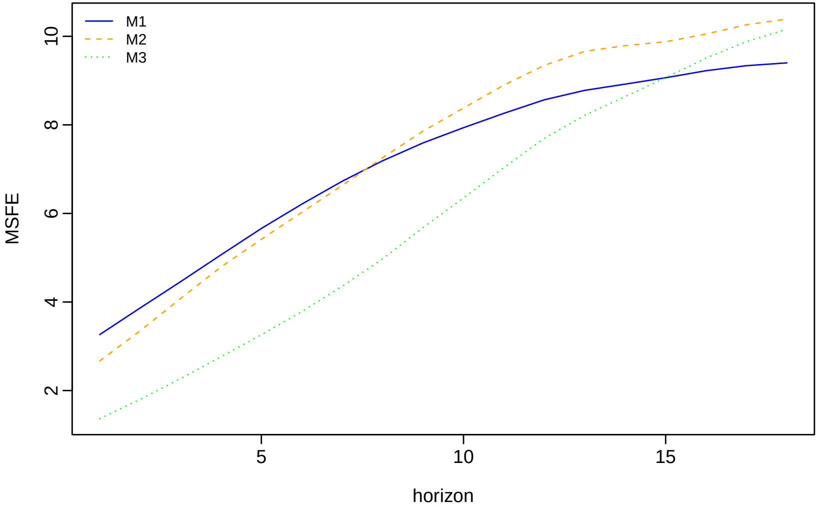

Figure 5: Canada underlying core inflation post-COVID MSFE at horizon h

CPI-Common-M3

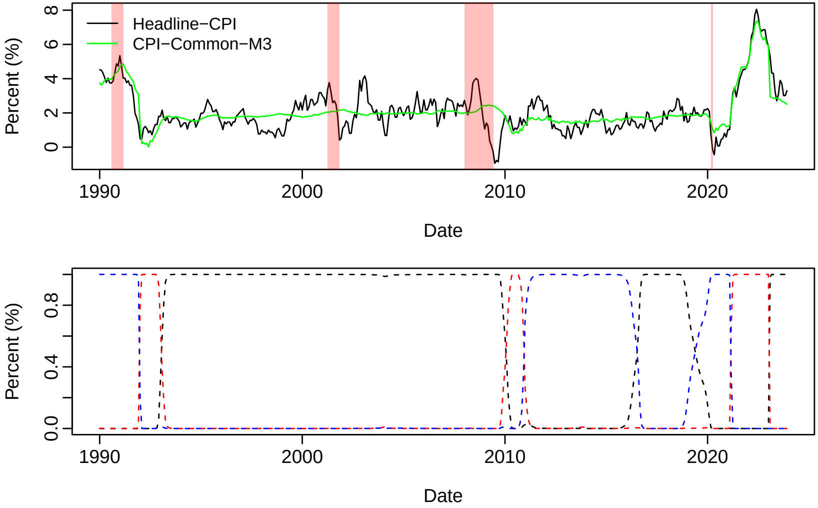

Figure 6: Canada underlying core inflation with $M = 3$ Jan-1990 to Dec-2023

Regime Transition Probabilities & Correlations

$$\mathbf{P} = \begin{bmatrix} 0.99 & 0.04 & 0.01 \\ 0.00 & 0.93 & 0.02 \\ 0.01 & 0.03 & 0.97 \end{bmatrix} \qquad \phi = [0.71, 0.10, 0.19]$$

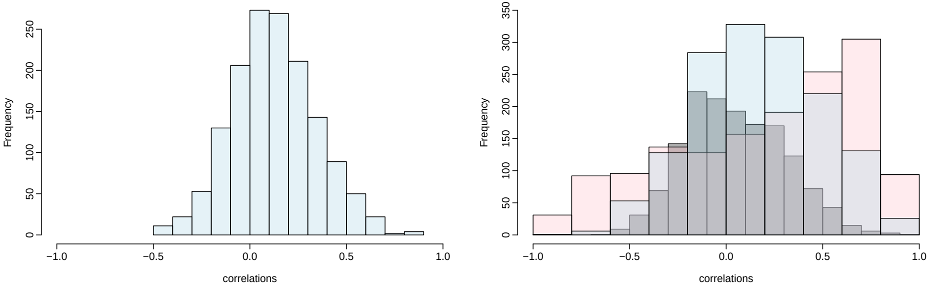

Figure 7: Correlation for Full Sample & Each Regime Jan-1990 to Dec-2023

Results

- Three regimes:
  - low, stable inflation regime (**black**) where $\mu_1 = 1.95$ & $\sigma_1^2 = 0.06$
  - high, non-stable inflation regime (red) where $\mu_2 = 3.03$ & $\sigma_2^2 = 6.38$
  - low, less stable inflation regime (blue) where $\mu_3 = 2.09$ & $\sigma_3^2 = 1.19$
- transition probabilities suggest all regimes are persistent
- correlations differ across regimes
- revision improve, especially during the rising inflation period
- Model with three regimes provides better forecasts of headline inflation up to one year out-of-sample

Hypothesis test for multiple structural breaks

Estimate and test structural break dates using least squares procedure described in Baltagi et al. (2021)

Table 3: Breaks in CPI-Common from Jan-1990 to Dec-2023

ϵT	Dmax ($M = 4$)		$l \| l+1$		
	UDmax	WDmax	F(1|2)	F(2|3)	F(3|4)
6	37.67**	37.67**	46.54**	46.81**	10.28
12	37.67**	37.67**	46.54**	9.21	9.25
24	36.15**	36.15**	44.03**	9.9	5.51

CPI-Common with multiple structural breaks

Break dates: 1991-06 (BoC adopted inflation-control target)
2022-03 (Rise in inflation)
2023-04 (inflation normalizing)

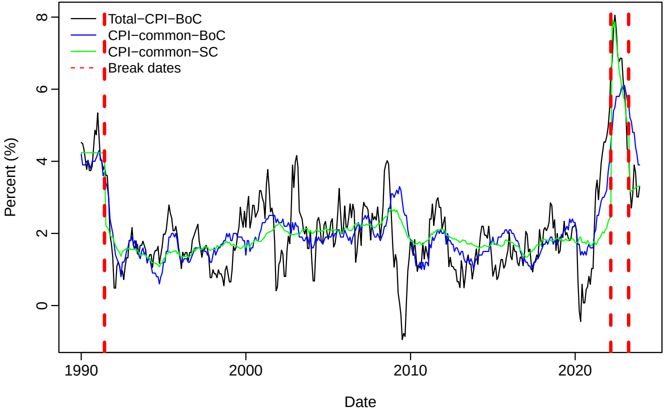

Figure 8: Correlation for Full Sample & Each Regime Jan-1990 to Dec-2023

Conclusion

- This paper proposes a new underlying core inflation indicator that
  1. has desirable features of a core inflation indicator
  2. is robust to abrupt changes
  3. reduces/mitigates revisions
- Markov switching approach is useful for real-time purposes and as short-term guide for monetary policy
- Structural change approach can eliminate revisions in some cases, but is an off-line method
- Canadian data is used to showcase value of new indicator
- US application (work in progress)

Conclusion

Thank you!

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Time-varying Inflation

Previous studies have considered structural change and Markov switching when modelling inflation (see Stock and Watson (2002); Perron et al. (2020); Amisano and Fagan (2013)) or time-varying parameters when estimating trend inflation (see Stock and Watson (2007); Stock and Watson (2016)).

In all cases, authors find evidence suggesting that inflation should be modelled using a time-varying framework.

Detecting multiple structural breaks

Multiple structural breaks in factor models:

- Baltagi et al. (2021) propose least-squares estimator of break dates and supF tests (like Bai and Perron (1998))
- Duan et al. (2022) propose a QML estimator and LRT (like Qu and Perron (2007))

Features:

- flexibility of structural change methods
- valid hypothesis testing procedures to determine number of breaks
- off-line detection method
  - depends on ϵ (determines the min length of regime); need to be $\epsilon \times T$ observations into the new regime to properly identify most recent break date
- cannot distinguish between breaks in factor loadings and breaks in factor variance

Real-time Results

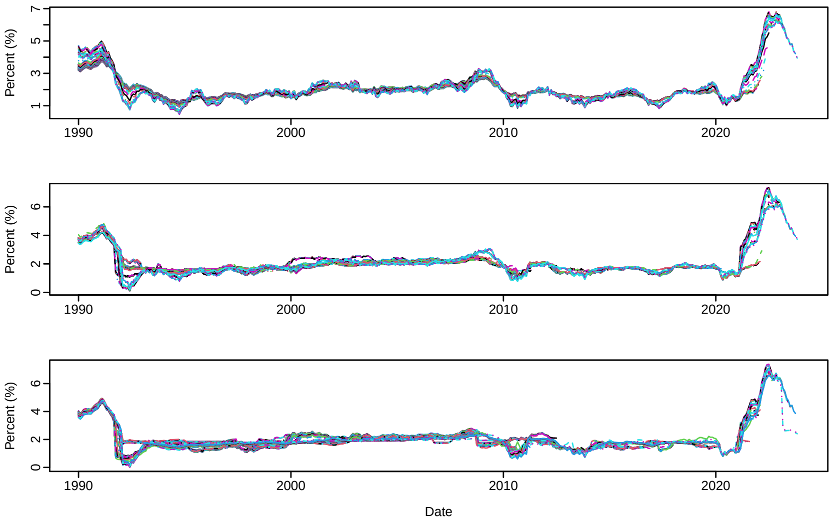


Figure 9: Canada Revisions CPI-Common Jan-1990 to Dec-2023

Core inflation indicator with structural breaks

$$X_t = \lambda_j F_t + e_t, \text{ if } z_t = j, \text{ for } t = 1, \dots, T \tag{8}$$

- $\bar{I}_\kappa$ & $\overline{\tilde{F}}_\kappa$ are the constant and estimated common factors for each regime (partitioned by $\hat{T}_\kappa$)
- assumption that α is subject to same changes as common factor(s) can be tested
- model is robust to changes in (1) mean inflation (i.e. α), (2) common factor and (3) the sensitivity to the common factor (i.e., β), which contribute to large revision (Sullivan (2022))
- predicted values $\hat{\Pi}$ are the resulting underlying core inflation indicator with structural breaks

CPI-Common with multiple structural breaks

Break dates: 1991-06 (BoC adopted inflation-control target)
2022-02 (Rise in inflation)

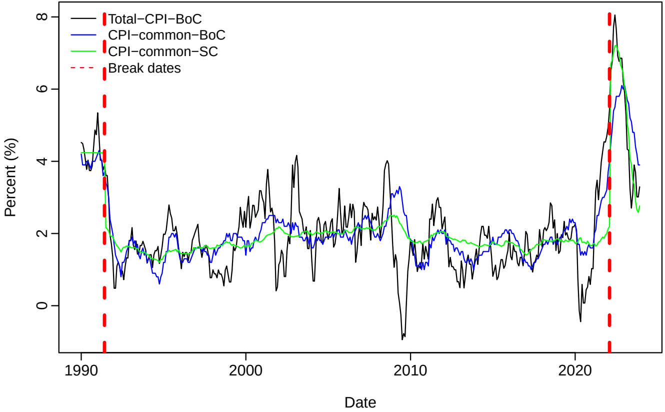


Figure 10: Correlation for Full Sample & Each Regime Jan-1990 to Dec-2023

Methodology

In matrix form:

$$X_\kappa = F_{0\kappa}\Lambda_0' + F_{-0\kappa}\Lambda_\kappa' + E_\kappa, \qquad t = T_{\kappa-1}+1,\ldots,T_\kappa \tag{9}$$

Define $\Lambda_{0,\kappa} = (\Lambda_0, \Lambda_\kappa)$. Baltagi et al. (2017) and Baltagi et al. (2021) show that there is an equivalent representation with stable loadings, Γ, and $\bar{r}$ pseudo factors g_t

$$X_\kappa = F_\kappa \Lambda_{0,\kappa}' + E_\kappa = F_\kappa R_\kappa' \Gamma' + E_\kappa = G_\kappa \Gamma' + E_\kappa \tag{10}$$

since $\Lambda_{0,\kappa} = \Gamma R_\kappa$ where R_κ is a $\bar{r} \times r$ selection matrix.

Detecting breaks

Baltagi et al. (2021) propose least-squares estimator of break dates and a supF test (like Bai and Perron (1998)) while Duan et al. (2022) propose a QML estimator and LRT (like Qu and Perron (2007)). Test stat for the former is

$$\sup_{(\tau_1,\ldots,\tau_l)\in\Lambda_\epsilon} F_{NT}\left(\tau_1,\ldots,\tau_l;\frac{\tilde{r}(\tilde{r}+1)}{2}\right) \tag{11}$$

$$F_{NT}\left(\tau_1,\ldots,\tau_l;\frac{\tilde{r}(\tilde{r}+1)}{2}\right) = \frac{2}{l\tilde{r}(\tilde{r}+1)}\left[SSNE_0 - SSNE(T_1,\ldots,T_l)\right] \tag{12}$$

Null distribution has the same form as Bai and Perron (1998) and Bai and Perron (2003).

Detecting breaks

As in Bai and Perron (1998), Baltagi et al. (2021) propose a UDmax and WDmax test that are used to test up to an unknown upper limit L number of breaks

$$\text{UDmax} = \max_{1 \leq l \leq L} \underset{(\tau_1,\dots,\tau_l)\in\Lambda_\epsilon}{sup} F_{NT}\left(\tau_1,\dots,\tau_l;\frac{\tilde{r}(\tilde{r}+1)}{2}\right) \tag{13}$$

$$\text{WDmax} = \max_{1 \leq l \leq L} \frac{c(\nu,\alpha,1)}{c(\nu,\alpha,l)} \underset{(\tau_1,\dots,\tau_l)\in\Lambda_\epsilon}{sup} F_{NT}\left(\tau_1,\dots,\tau_l;\frac{\tilde{r}(\tilde{r}+1)}{2}\right) \tag{14}$$

where $\nu = \frac{\tilde{r}(\tilde{r}+1)}{2}$ and the sequential $\text{F}(l|l+1)$ test that can be used to determine the appropriate number of breaks.

$$\text{F}(l|l+1) = SSNE(T_1,\dots,T_l) - \min_{1\leq\iota\leq l+1\in\Lambda_{\iota,\epsilon}} SSNE(T_1,\dots,T_{\iota-1},\tau,T_\iota,\dots,T_l) \tag{15}$$

Markov Process

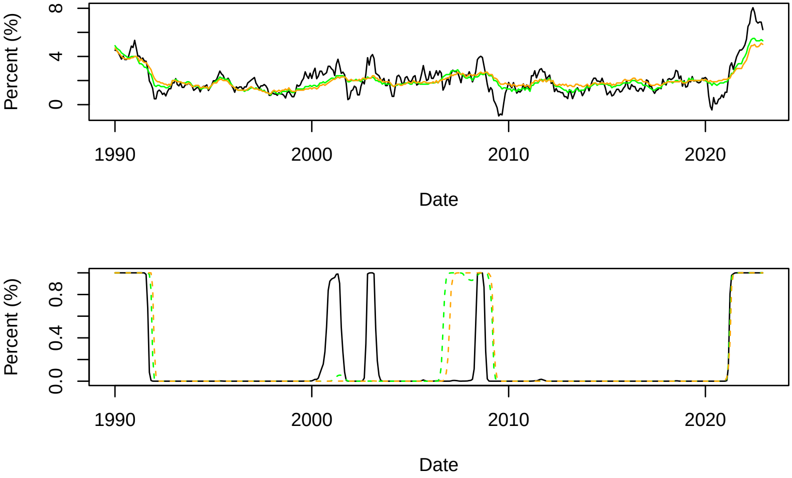
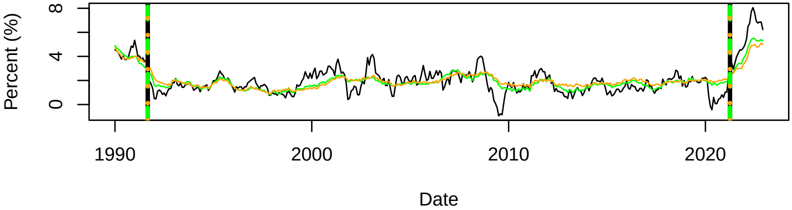


Figure 11: Other Inflation & Core inflation indicators with their Markov Process (S_t) Jan-1990 to Dec-2022 (**black**: all, green: trim, orange: med)

Structural Change



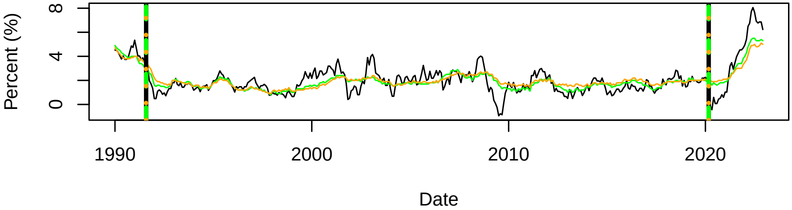


Figure 12: Other Inflation & Core inflation indicators with their break dates Jan-1990 to Dec-2022 (top: based on mean only, bottom: based on variance only; **black**: all, green: trim, orange: med)